

# Dynamic response of a tower-pier system on viscoelastic foundation with frictional interface

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The response of a bridge tower-pier system resting on elastic soil, with a sliding foundation obeying Coulomb's friction law, is studied. The stiffness and damping of the supporting soil is fully taken into account. The action of the suspension bridge cables is equivalent to a horizontal elastic restraint at the top of the tower. The differential equations of motion of the pier are formulated for its horizontal displacement and rocking angle. The two possible modes of horizontal motion of the pier, the 'arrested' and the 'sliding' motion, are taken into account. The transitions between these two modes are considered during numerical integration. A parametric study is carried out with emphasis on the residual slippage and the developed moment in the pier foundation due to rocking. This study was motivated by the need to understand the potential seismic response of a large suspension bridge proposed for the Rion-Antirrion straits, in Greece. Copyright © 1996 Elsevier Science Ltd.

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# 1. Introduction

It is well known that the towers are the most important structural members of suspension bridges when considering protection against seismic excitations. This is particularly so if the suspension length is great and the towers are founded on relatively soft soil. Research on the earthquake resistance analysis of suspension bridge towers with massive pier-foundation systems has been published in Japan and the USA<sup>1-3</sup>.

A simplified approach to determine the seismic response of a suspension bridge tower is presented in Reference 4, where the flexibility of the underlying soil is taken into account, while the tower behaves as a distributed flexural beam.

On the other hand significant research has been done on systems including Coulomb friction. Spring-mass vibratory systems (oscillators) with or without viscous friction having a Coulomb (dry) friction contact have been studied<sup>5-9</sup>. Analytical, numerical or combined analytical-numerical solutions of the equations of motion are found describing the

arrested or slip motion of the oscillators. Relations between the kind of motion and the governing pparameters are obtained theoretically and experimentally. Results are presented in nondimensional form as magnification factors versus excitation to natural frequency ratios etc.

If the Coulomb frictional force is represented by a Fourier series, an analytic solution of the equation of motion of the oscillator can be obtained, assuming that only slip motion occurs, without stops during any portion of the oscillation<sup>10</sup>.

The asymptotic stability of the steady-state motions of the oscillator has been studied, considering a static coefficient of friction different from the dynamic one<sup>11</sup>.

An isolation mechanism that utilizes linear Coulomb friction, i.e. progressively increasing frictional resistance, has been investigated<sup>12-14</sup>. The mathematical model of this device is incorporated into the equations of motion which are solved analytically and numerically.

The effect of the vertical component of ground motion on the horizontal response of a Coulomb-type sliding system has been studied<sup>15</sup>, including the soil-foundation interaction. The results show that the effect of vertical motion is significant in the cases of harmonically excited foundations.

The response of a two-degrees-of-freedom oscillator supported on a Coulomb-type frictional interface, to harmonic base excitation, has been considered<sup>16</sup>. The analysis of this model furnishes interesting results related to the resonant frequencies and the peak response, which are useful in the area of vibration isolation of more complex structural or mechanical systems.

Comparative studies of the effectiveness of different base isolators including Coulomb friction have been carried out<sup>17-19</sup>. Several sensitivity analyses are also worked out. The performances of the isolation devices under a variety of conditions are evaluated and discussed.

In the present work, we study the behaviour against horizontal ground excitation of a suspension bridge tower-pier system, taking into account that at the foundation interface, the shear friction force obeys Coulomb's friction law, permitting alternating 'arrested' and 'slipping' modes. During an arrested mode the pier remains stuck onto the underlying soil and no relative displacement of the pier with respect to the soil takes place. On the other hand during a slipping mode the pier slides onto the soil and a constant shear force is developed in the foundation surface opposing the relative slipping between the pier and the soil. Since the mass of the tower is indeed negligible compared to the mass of the pier, the tower is modelled as a vertical massless cantilever beam, perfectly fixed onto the pier. The horizontal constraint at the top of the tower due to the presence of the suspension cables is replaced by a horizontal spring of known stiffness. The action of the cables and the deck upon the bridge tower are replaced by an equivalent vertical force acting on top of the tower. Furthermore the influence of the supporting soil is modelled rigorously through the frequency-dependent rocking and swaying impedances (stiffnesses and dashpots), for a circular foundation on the surface of a homogeneous half-space. The differential equations of motion of the pier are formulated and integrated numerically.

This work was motivated by the study for the seismic design of a suspension bridge for Rion-Antirrion<sup>20</sup>. Three factors contributed to the need for unusually large (even by suspension-bridge standards) piers for that bridge: firstly, the large water depth (nearly 60 m) and the design requirement for the pier to withstand a full-speed collision with a tanker; secondly, the poor soil conditions (relatively-loose soil layers down to at least 50 m depth; bedrock located at depths exceeding 200 m); and finally, the very strong seismic design ground shaking (pga = 0.525g, PSA (T = 1 s) = 0.79g).

It turned out that a simple model of the soil-pier-towersuspended-bridge system, such as the one studied herein, was not only a very practical solution (in view of the extremely limited time that was available) but also constituted a reasonably good approximation to the exact problem. This was rather easy to explain, since the mass of the pier was two orders of magnitude greater than the mass of superstructure carried by the pier.

The results presented herein are for horizontal excitation only, the rocking component of excitation being insignificant, since vertical shear wave excitation is considered.

In this work the vibration was considered in the longitudinal direction only. The choice was deliberate: this is, potentially, the most disadvantageous direction, in view of the developing restraining force from the cables. Lateral excitation and motion can be and was, in fact, studied independently; moreoever, it could be readily 'recovered' from the presented analysis.

#### 2. Structural model

In Figure 1a the bridge tower of height h is depicted, fixed onto the pier of total height L + z. Denote by C the centre of mass of the pier. The action of the cables at the top of the bridge tower, has been replaced by the spring of stiffness  $k_e$ , as well as by a constant vertical compressive force P, acting on top of the tower.

The pier rests on soil modelled as a homogeneous elastic half-space<sup>21</sup>. Figure 1b presents the structural model, and shows the earthquake motion  $x_g(t)$ , the swaying stiffness  $k_s$  and damping  $c_s$  at the soil-foundation interface, the possibility of sliding of the pier according to Coulomb's friction law, as well as the stiffness  $k_{\psi}$  and damping  $c_{\psi}$  against rocking of the pier. Any dynamic rotation  $\theta(t)$  of the pier results in a reacting moment

$$M_{\rho} = k_{\psi} \,\theta + c_{\psi} \,\dot{\theta} \tag{1}$$

The contact between the pier and the soil obeys Coulomb's friction law. Hence, during horizontal oscillation the pier remains stuck to the soil as long as the shear force F at the interface satisfies

$$F < \mu P_t \tag{2}$$

where  $\mu$  is the friction coefficient, and

$$P_t = P + m_p g \tag{3}$$

is the total normal reaction developed at the foundation surface,  $m_p$  is the pier mass.

When equation (2) ceases to hold true, the pier starts sliding and the shear force at the interface takes the value

$$F = \mu P_t \operatorname{sgn}(u_s - u) \tag{4}$$

where u and  $u_s$  are the velocities of the base of the pier and the underlying soil, respectively.

The sliding between pier and soil ceases and the pier again remains stationary on the underlying soil at time  $t^*$  such that

$$u(t^*) = u_s(t^*) \tag{5a}$$

$$F(t^*) < \mu P_t \tag{5b}$$

where the friction shear force  $F(t^*)$  in equation (5b) is evaluated under the assumption that the pier is stuck onto soil.

A displacement x of the pier base and a rotation  $\theta$  of the pier, would cause a horizontal displacement

$$y = x - \theta(h + L + z_c) \tag{6}$$

of the top of the tower, if it were not constrained by the spring (*Figure 1b*). But the action of the spring results in the development of the spring force S, producing shortening of the spring

$$x_1 = S/k_e \tag{7}$$



Figure 1 (a) Suspension bridge pier-foundation system; (b) structural model

as well as deflection of the top of the tower

$$x_2 = S/k_h \tag{8}$$

where

$$k_h = \frac{P}{h} \left( \frac{\tan a}{a} - 1 \right)^{-1} \tag{9a}$$

is the bending stiffness of an axially loaded cantilever  $^{22}, \ensuremath{\mathsf{and}}$  and

$$a = \left(\frac{Ph^2}{EI}\right)^{1/2} \tag{9b}$$

where El denotes the tower flexural rigidity. Obviously

$$y = x_1 + x_2 \tag{10}$$

and hence the combined stiffness  $k_t$  of the tower and the spring is given by

$$1/k_t = (1/k_e) + (1/k_h) \tag{11}$$

Finally, the shear force on the bridge tower is

$$S = k_t \left[ x - \theta \left( h + L + z_c \right) \right]$$
(12)

For the model to be presented in this work, the following

data will be used, taken from an actual long suspension bridge

$$h = 176 \text{ m}$$

$$L = 49 \text{ m}$$

$$z_c = 16 \text{ m}$$

$$P = 400.00 \text{ kN}$$

$$EI = 9.59 \times 10^{11} \text{ kN m}^2$$

$$k_e = 700.000 \text{ kN/m}$$

$$m_p = 335.000 \text{ Mg}$$

$$I_p = 1.9 \times 10^8 \text{ Mg m}^2$$
(13)

where  $m_p$  is the pier mass, and  $I_p$  is the pier moment of inertia about its centre of gravity C.

$$k_s = 43.000.000 \text{ kN/m}$$
  
 $c_s = 2.180.000 \text{ kN s/m}$   
 $k_{\psi} = 1.0091 \times 10^{11} \left(1 - \frac{\omega}{30}\right) \text{ kN m}$   
 $c_{\psi} = 5.3608 \times 10^9 (1 - e^{-0.0333 \, \omega^{1.70}}) \text{ kN m/s}$ 

where  $\omega$  is the angular velocity of the harmonic part of the ground motion. The above k and c values are for a circular foundation of radius r = 50 m on a soil with  $V_s = 300$  m/s (see Reference 21).

## 3. The equations of motion

Figure 2a depicts all the actions on the displaced pier. x and  $\theta$  denote the displacement of the base of the pier and its rotation, respectively. The displacement of the centre of mass of the pier is then

$$x_c = x - \theta \, z_c \tag{14}$$



(a)

Figure 2 (a) Pier dynamic equilibrium; (b) horizontal force equilibrium at foundation interface

The shear force of the bridge tower is given by equation (12) and the overturning moment at the base of the tower is

$$M_t = hk_t \left[ x - \theta (h + L + z_c) \right]$$
(15)

Figure 2b shows the equilibrium of the horizontal forces acting at the foundation interface. F is the reaction from the pier given alternately by equation (2) or equation (4). It is obvious that

$$F = -c_s \dot{x}_s - k_s x_s + c_s \dot{x}_g + k_s x_g \tag{16}$$

In the following, the equations of the horizontal motion as well as of the rotation of the pier are formulated separately for each possible mode.

#### 3.1. Stuck motion

If  $x_s$  is the displacement of the soil surface immediately below the foundation

$$x = x_s + \Delta x \tag{17}$$

gives the horizontal displacement of the pier base, where  $\Delta x$  is the sliding between the pier and the supporting soil which has accumulated at the time that the particular stuck motion starts.

The equilibrium of the horizontal forces acting on the pier, is expressed by

$$F - m_p \left( \ddot{x} - \ddot{\theta} z_c \right) - k_t \left[ x - \theta (h + L + z_c) \right] = 0$$

from which

$$F = -m_p z_c \ddot{\theta} - k_t (h + L + z_c) \theta + m_p \ddot{x}_s + k_t x_s + k_t \Delta x$$
(18)

It is assumed, of course, that the above value of F satisfies the inequality of equation (2). Moment equilibrium gives

$$I_{p}\ddot{\theta}+c_{\psi}\dot{\theta}+\left[k_{\psi}+k_{t}(h+L+z_{c})(h+L)\right]$$
$$-P(h+L+z_{c})\frac{k_{h}}{k_{e}+k_{h}}-m_{p}gz_{c}\theta$$
$$+\left[-k_{t}(h+L)-P\frac{k_{e}}{k_{e}+k_{h}}\right]x-Fz_{c}=0$$

from which, using equations (16b) and (17)

$$\ddot{\theta} = -\frac{c_{\psi}}{I_{p}}\dot{\theta} + \left[-\frac{k_{\psi} + k_{t}(h + L + z_{c})(h + L) - P(h + L + z_{c})\frac{k_{h}}{k_{e} + k_{h}} - m_{p}gz_{c}}{I_{p}}\right]$$
$$\theta - \frac{c_{s}z_{c}}{I_{p}}\dot{x}_{s} + \left[\frac{k_{t}(h + L) + P\frac{k_{e}}{k_{e} + k_{h}} - k_{s}z_{c}}{I_{p}}\right]x_{s} + \frac{c_{s}z_{c}}{I_{p}}\dot{x}_{g} + \frac{k_{s}z_{c}}{I_{p}}x_{g}$$

$$+\left[\frac{k_t(h+L) + P \frac{k_e}{k_e + k_h}}{I_p}\right]\Delta x = 0$$
(19)

Equating the right-hand sides of equations (16b) and (18) and substituting in the resulting equation the expression for  $\ddot{\theta}$  as given by equation (19), we obtain

$$\begin{split} \vec{x}_{s} &= \left[ -\frac{c_{\psi}z_{c}}{I_{p}} \right] \dot{\theta} + \left[ \frac{k_{t}\left(h+L+z_{c}\right)}{m_{p}} - z_{c} \right] \\ \frac{k_{\psi} + k_{t}\left(h+L+z_{c}\right)\left(h+L\right) - P(h+L+z_{c})\frac{k_{t}}{k_{e} + k_{h}} - m_{p}gz_{c}}{I_{p}} \right] \theta \\ &+ \left[ -\frac{c_{s}}{m_{p}} - \frac{c_{s}z_{c}^{2}}{I_{p}} \right] \dot{x}_{s} \\ &+ \left[ -\frac{k_{t} + k_{s}}{m_{p}} + z_{c}\frac{k_{t}(h+L) + P\frac{k_{e}}{k_{e} + k_{h}} - k_{s}z_{c}}{I_{p}} \right] x_{s} \\ &+ \left[ \frac{c_{s}}{m_{p}} + \frac{c_{s}z_{c}^{2}}{I_{p}} \right] \dot{x}_{s} + \left[ \frac{k_{s}}{m_{p}} + \frac{k_{s}z_{c}^{2}}{I_{p}} \right] x_{s} \\ &+ \left[ -\frac{k_{t}}{m_{p}} + z_{c}\frac{k_{t}\left(h+L\right) + P\frac{k_{e}}{k_{e} + k_{h}}}{I_{p}} \right] \Delta x \end{split}$$

$$(20)$$

From equation (19) it is evident that the rocking eigenfrequency of the pier is given by

$$\omega_{0} = \left[ -\frac{k_{\psi} + k_{\iota} (h + L + z_{c}) (h + L) - P(h + L + z_{c}) \frac{k_{h}}{k_{e} + k_{h}} - m_{p} g z_{c}}{I_{p}} \right]^{1/2}$$
(21)

# 3.2. 'Sliding' motion

When the pier slides on the soil, the shear force at the foundation surface is given by equation (4), which now is written as

$$F = \mu P_t \operatorname{sgn}(\dot{x}_s - \dot{x}) \tag{22}$$

In this case equilibrium of moments gives

$$I_{p} \ddot{\theta} + c_{\psi}\dot{\theta}$$

$$+ \left[k_{\psi} + k_{t} \left(h + L + z_{c}\right) \left(h + L\right) - P\left(h + L + z_{c}\right) \frac{k_{h}}{k_{e} + k_{h}} - m_{p}gz_{c}\right]$$

$$\theta + \left[-k_{t} \left(h + L\right) - P\frac{k_{e}}{k_{e} + k_{h}}\right]x - Fz_{c} = 0 \qquad (23a)$$

which finally becomes

 $\ddot{\theta} = - \, \frac{c_\psi}{I_v} \, \dot{\theta}$ 



*Figure 3* Ground motion: (a) displacement; (b) velocity; (c) acceleration

$$+ \left[ -\frac{k_{\psi} + k_{r} (h + L + z_{c}) (h + L) - P (h + L + z_{c}) \frac{k_{h}}{k_{e} + k_{h}} - m_{\rho} g z_{c}}{I_{\rho}} \right] \theta + \left[ \frac{k_{r} (h + L) + P \frac{k_{e}}{k_{e} + k_{h}}}{I_{\rho}} \right] x - \frac{\mu P_{r} z_{c}}{I_{\rho}} \operatorname{sgn}(\dot{x} - \dot{x}_{s})$$
(23b)

Equilibrium of the horizontal forces gives

$$sgn(\dot{x}_s - \dot{x}) \ \mu P_t - m_p \ (\ddot{x} - \ddot{\theta} z_c)$$

$$-k_t \ [x - \theta \ (h + L + z_c)] = 0$$
(24a)

from which

$$\ddot{x} = \frac{k_t \left(h + L + z_c\right)}{m_p} \theta - \frac{k_t}{m_p} x - \frac{\mu P_t}{m_p} \operatorname{sgn}(\dot{x} - \dot{x}_s) + z_c \ddot{\theta}$$
(24b)

Substituting equation (23b) into the above equation we obtain

$$\begin{split} \ddot{x} &= -\frac{C_{\psi}z_{c}}{I_{p}} \dot{\theta} + \left[ \frac{k_{i} (h + L + z_{c})}{m_{p}} \right] \\ &- z_{c} \frac{k_{\psi} + k_{i} (h + L + z_{c}) (h + L) - P (h + L + z_{c}) \frac{k_{h}}{k_{e} + k_{h}} - m_{p}gz_{c}}{I_{p}} \right] \theta \\ &+ \left[ -\frac{k_{i}}{m_{p}} + z_{c} \frac{k_{i} (h + L) + P \frac{k_{e}}{k_{e} + k_{h}}}{I_{p}} \right] x \\ &+ \left[ -\frac{\mu P_{i}}{m_{p}} - \frac{\mu P_{i}z_{c}^{2}}{I_{p}} \right] \operatorname{sgn}(\dot{x} - \dot{x}_{s}) \end{split}$$
(24c)

In the case of 'sliding motion' equation (16a) produces

$$c_s \dot{x}_s - c_s \dot{x}_g + k_s x_s - k_s x_g - \mu P_t \operatorname{sgn}(\dot{x} - \dot{x}_s) = 0$$

from which

$$\dot{x}_s = -\frac{k_s}{c_s}x_s + \dot{x}_g - \frac{k_s}{c_s}x_g + \frac{\mu P_t}{c_s}\operatorname{sgn}(\dot{x} - \dot{x}_s)$$
(25a)

and

$$\ddot{x}_s = -\frac{k_s}{c_s}\dot{x}_s + \ddot{x}_g - \frac{k_s}{c_s}\dot{x}_g$$
(25b)

for the velocity and acceleration of the soil on the surface of the foundation.



Figure 4 Pier response: (a) relative displacement; (b) relative velocity; (c) relative velocity; (d) acceleration; (e) frictional force at foundation interface; (f) rocking angle

## 4. Ground motion

In the foregoing (equations (16), (19), (20) and (25)) the horizontal ground displacement and its time derivatives must be known. For the purposes of the present work we have selected as ground displacement  $x_g(t)$ , velocity  $\dot{x}_g(t)$ and acceleration  $\ddot{x}_g(t)$  the functions plotted in Figures 3ac. Each of the quantities  $x_g(t)$ ,  $\dot{x}_g(t)$  and  $\ddot{x}_g(t)$  is composed of five parts. The first and second parts start the motion, the fourth and fifth end it, while the third is the main part of the motion, consisting of five sinusoidal cycles.

The period T and the amplitude  $x_{g_0}$  of each cycle of this part are the parameters chosen to achieve the desired ground excitation. It should be noticed that all three quantities  $(x_g, \dot{x}_g, \text{ and } \ddot{x}_g)$  start from, and end at, zero (at-rest) values.

### 5. Response and parametric study

Each of the systems of differential equations (19)-(20) and (23b)-(24c)-(25) along with the initial conditions

$$\begin{aligned} x(0) &= \dot{x}(0) = \dot{x}(0) = 0 \\ x_s(0) &= \dot{x}_s(0) = \ddot{x}_s(0) = 0 \\ \theta(0) &= \dot{\theta}(0) = \ddot{\theta}(0) = 0 \end{aligned}$$
(26)

governs the response of the pier during a 'stuck' motion or a 'slipping' motion, respectively. The transition from a 'stuck' to a 'slipping' motion and vice versa is ruled by the conditions given in equations (2) and (5). The unknowns are the functions x(t),  $x_s(t)$  and  $\theta(t)$ .

Employing the Runge-Kutta method, the above systems are integrated numerically. At the end of each integration step, the aforementioned conditions of separation or reattachment are checked, in order to continue the integration using the proper differential system.

Some results of the pier response evaluated with the above integration scheme, are shown in *Figures 4a-f.* Figure 4a presents the relative displacement  $x_r$  of the pier with respect to the ground for  $x_{g_0} = 0.1$  m and  $T_g = 0.8$  s. Figures 4b and 4c present the relative velocity  $\dot{x}_r$  of the pier with respect to the ground, for the cases of: (i) mild



Figure 5 Parametric study: complete solution x<sub>r.max</sub>, x<sub>r.min</sub>



Figure 6 Paramtric study: complete solution  $x_{r,fin}$ 



Figure 7 General behaviour of x<sub>r,max</sub>, x<sub>r,min</sub>

ground excitation ( $x_{g_0} = 0.1$  m and  $T_g = 0.8$  s); and (ii) severe ground excitation ( $x_{g_0} = 0.15$  m and  $T_g = 0.6$  s), respectively.

Figures 4d and 4e show the pier acceleration  $\ddot{x}$  and the developed frictional shear force F at the base, respectively, for  $x_{g_0} = 0.1$  m and  $T_g = 0.8$  s. Finally, in Figure 4f, one can see the variation of the rocking angle  $\theta(t)$  of the pier versus time, also for  $x_{g_0} = 0.1$  m and  $T_g = 1.0$  s. A parametric study of the pier motion is carried out to

A parametric study of the pier motion is carried out to investigate the influence of the ground motion amplitude  $x_{g_0}$  and angular velocity  $\omega_g$ , as well as of the friction coefficient  $\mu$ , upon the maximum, the minimum and the residual sliding of the pier with respect to the ground, as well as upon the maximum and minimum reacting moments  $M_p$ .

Figures 5a and 5b show the variations of maximum and minimum relative slippage  $x_{r,max}$  and  $x_{r,min}$  versus the ratio of ground frequency  $\omega_g$  over the tower-pier system eigenfrequency  $\omega_0$  (equation (19)) plotted for different values of the ground motion amplitude  $x_{g_0}$  ( $x_{g_0} = 0.10, 0.15, 0.20,$ 0.25) and different values of the friction coefficient  $\mu$  ( $\mu$ = 0.5, 1.00).



Figure 8 Parametric study: complete solution  $M_{p,max}$ ,  $M_{p,min}$ 

In Figure 5c the variation of  $x_{r,max}$  and  $x_{r,min}$  versus  $\omega_g/\omega_0$ , is presented for  $x_{g_0} = 0.15$  m and for different values of  $\mu$ .

Similarly Figures 6a and 6b depict the variations of the residual sliding  $x_{r,in}$  versus  $\omega_g/\omega_0$  for different values of  $x_{g_0}$  and different values of  $\mu$ . On the other hand, Figure 6c plots the variation of  $x_{r,fin}$  versus  $\omega_g/\omega_0$  for  $x_{g_0} = 0.15$  and for different values of  $\mu$ . Figure 7 summarizes the general trends in  $x_{r,max}$  and  $x_{r,min}$ , as will be discussed later on.

Figures 8a and 8b plot versus  $\omega_g/\omega_0$  the maximum and minimum reacting moments,  $M_{p,max}$  and  $M_{p,min}$ , nondimensionalized by the quantity  $\mu P_t z_c$ , where  $\mu P_t$  is the maximum frictional shear force at the foundation surface, for different values of  $\mu$  and for  $x_{g_0} = 0.10$  m and 0.20 m, respectively.

Furthermore, in order to evaluate the potential errors that would arise when certain popular 'simplifications' are introduced in the analysis, the results of *Figures 9* and *10* are included herein.

Figure 9 illustrates the effect of soil (material and

radiation) damping, expressed through  $c_s$  and  $c_{\psi}$ , by comparing the complete solution with the solution derived by ignoring both  $c_s$  and  $c_{\psi}$ . Figure 10 shows the effect of ignoring the rotational component of pier motion.

## 6. Discussion

In Figure 4a the alternating arrested and sliding modes of motion are shown for a moderate ground shaking. Comparing Figures 4b and 4c it can be seen that large values of  $x_{g_0}$  and  $\omega_g$ , i.e. severe ground excitation, prevent the existence of periods of stuck motion (relative velocity = 0 for long periods of time).

During slipping motion reversals of slip direction cause rapid changes in pier acceleration which otherwise, during stuck motion, follows the ground acceleration (*Figure 4d*).

The frictional force following Coulomb's friction law exhibits, understandably, a step-function behaviour (*Figure* 4e) during the sliding modes of motion.







Figure 9 Parametric study:  $c_s = c_{\psi} = 0$ 

The time variation of the rocking angle  $\theta$  (Figure 4f) is the superposition of a main sinusoidal rotation with the frequency of the ground motion and a secondary sinusoidal rotation with the rocking eigenfrequency of the system. Rocking decreases asymptotically after the end of the ground motion.

From the results of the parametric study (*Figures* 5a-c) one can see first that the maximum slippage of the pier,

 $x_{r,\max}$ , despite some random local abnormalities, exhibits a general behaviour as shown in *Figures 7a* and 7b. In both these figures, for values of  $\omega_g$ , close to zero, there is an interval of purely arrested motion. After some value of  $\omega_g$  slipping intervals interfere with the motion and  $x_{r,\max}$  increases with  $\omega_g$  up to the value  $\omega_g/\omega_0 = 1$ , where  $x_{r,\max}$  falls rapidly to a low value which is kept constant for all  $\omega_g/\omega_0 \ge 1$ . The curve  $x_{r,\max}$  versus  $\omega_g/\omega_0$  is an increasing



Figure 10 Parametric study: no rocking

function of the parameter  $x_{g_0}$  (amplitude of the ground motion) as expected. The value of  $\omega_g/\omega_0$  for which  $x_{r,max}$ falls to low values is a decreasing function of  $x_{g_0}$  (Figure 7a) and an increasing function of  $\mu$  (Figure 7b). Similarly the border between purely arrested motion and motion with slipping intervals is also a decreasing function of the ground motion amplitude  $x_{g_0}$  (Figure 7a) and an increasing function of the friction coefficient  $\mu$  (Figure 7b).

For  $x_{r,\min}$ , (Figures 5a-c) there are also similar local

abnormalities around the  $\omega_g/\omega_0$  values of 0.5 and 1. For  $\omega_g/\omega_0 > 1$ ,  $x_{r,\min}$  attains a constant value, such that  $|x_{r,\min}|$  is an increasing function of  $x_{g_0}$ .

Coming now to the dimensionless moments  $M_{p,\max}$ ,  $M_{p,\min}$  developed on the foundation surface (*Figures 8a* and 8b), one observes almost zero moments for  $\omega_g/\omega_0$  very small, where the motion is very slow. The largest values of the moment occur in the interval  $0.30 < \omega_g/\omega_0 < 1.30$ , i.e. around the elastic resonance ( $\omega_g = \omega_0$ ).

At higher values of  $\omega_g/\omega_0$ , the moment remains nearly constant, and at even higher values,  $\omega_g/\omega_0 > 2$ , where  $k_{\psi} < 0$  (equation (13)),  $|M_{p,\min}|$  increases rapidly.

For constant  $x_{g_0}$  the dimensionless moments decrease with  $\mu$ , which indicates that higher frictional capacity at the foundation-soil interface ensures better isolation and protection against the development of reacting moments. Finally, for constant  $\mu$  the dimensionless moment is evidently an increasing function of  $x_{g_0}$ .

Figure 9 plots results of the parametric study investigating the special case  $c_s$ ,  $c_{\psi} = 0$ . As expected, due to lack of damping,  $x_{r,max}$ ,  $x_{r,min}$  and  $x_{r,fin}$  are larger and compared with the complete case  $(c_s, c_{\psi} \neq 0)$  less smooth.

Explicitly larger values of the dimensionless moments  $M_{p,\text{max}}$  and  $M_{p,\text{min}}$  appear in Figure 9c, since lack of damping  $(c_s, c_{\psi} = 0)$  permits a stronger rocking.

Finally, Figure 10 demonstrates the behaviour of the pier, if the latter is (spuriously) 'fixed' against rotation  $(k_{\psi} = \infty)$ . Compared with the complete case, the development of some higher sliding values around the resonance area  $\omega_g/\omega_0 = 1$ , (as expected since rocking absorbed some energy) should be noticed.

As for the behaviour of the dimensionless rocking moments (*Figure 10c*), when motion has been arrested ( $0 < \omega_g/\omega_0 < 0.4$ ), rotation has apparently very little effect (the two curves nearly coincide). Once sliding motion starts ( $\omega_g/\omega_0 > 0.4$ ), the dimensionless moments (in the no rocking case) are constant and equal to 1, since apparently the shear force  $\mu P_t$  at the foundation interface is constant, with lever arm  $z_c$ . By contrast, in the complete solution (with rotation taking place) the moments are significantly amplified by a factor of almost three.

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